Analyse vectorielle

Gradient :

$$\boxed{df = \overrightarrow{\text{grad}} f \cdot \overrightarrow{dl}}$$

coordonnées cartésiennes :

$$\overrightarrow{\text{grad}}\, V = \left| \begin{array}{l} \dfrac{\partial V}{\partial x} \\[2mm] \dfrac{\partial V}{\partial y} \\[2mm] \dfrac{\partial V}{\partial z} \end{array} \right.$$

on peut définir seulement en coordonnées cartésiennes l'opérateur "nabla" noté $\vec{\nabla}$:

$$\vec{\nabla} = \left| \begin{array}{l} \dfrac{\partial}{\partial x} \\[2mm] \dfrac{\partial}{\partial y} \\[2mm] \dfrac{\partial}{\partial z} \end{array} \right. \qquad \text{et donc } \underline{\overrightarrow{\text{grad}}\, V = \vec{\nabla} V}$$

Divergence :

coordonnées cartésiennes :

$$\text{div}\, \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\underline{\text{div}\, \vec{A} = \vec{\nabla} \cdot \vec{A}} = \left| \begin{array}{l} \dfrac{\partial}{\partial x} \\[2mm] \dfrac{\partial}{\partial y} \\[2mm] \dfrac{\partial}{\partial z} \end{array} \right. \cdot \left| \begin{array}{l} A_x \\[2mm] A_y \\[2mm] A_z \end{array} \right.$$

Rotationnel :

$$\overrightarrow{\text{rot}}\, \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{u}_z$$

coordonnées cartésiennes :

$$\overrightarrow{\text{rot}}\, \vec{A} = \vec{\nabla} \wedge \vec{A} = \left| \begin{array}{l} \dfrac{\partial}{\partial x} \\[2mm] \dfrac{\partial}{\partial y} \\[2mm] \dfrac{\partial}{\partial z} \end{array} \right. \wedge \left| \begin{array}{l} A_x \\[2mm] A_y \\[2mm] A_z \end{array} \right. = \left| \begin{array}{l} \dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z} \\[2mm] \dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x} \\[2mm] \dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y} \end{array} \right.$$

$$\frac{\partial}{\partial x} \qquad A_x$$

Laplacien scalaire : noté $\Delta V = \mathrm{div}(\overrightarrow{\mathrm{grad}}\, V)$

coordonnées cartésiennes : $\Delta V = \vec{\nabla} \cdot (\vec{\nabla} V)$

$$\Delta V = \begin{vmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{vmatrix} \cdot \begin{vmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{vmatrix} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Laplacien vectoriel : $\vec{\Delta} \vec{V} = \begin{vmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{vmatrix}$